

Six Things You Should Not Do With CIELAB

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Wax Technology

CIELAB has served us well. It has provided a reliable and unambiguous way for a brand owner to uniquely specify a target color. It has enabled spectrophotometers to assist through the workflow by providing standardization of color. ICC profiles and G7 are based on CIELAB. By using color difference formulas based on CIELAB, we have an objective way to determine if a printed product is in spec.

But, CIELAB is not without foibles. Its development occurred without a full knowledge of how it would be used. Some of the decisions that went into CIELAB have had far-reaching and underappreciated consequences. As a result, there are some expectations that we have for CIELAB that are questionable.

This paper describes a number of the problems with CIELAB by looking at where it does not perform well. There is a subtle undercurrent in the paper that the industry needs to start considering how to replace CIELAB with a more modern color space that does share CIELAB's issues.

Things you should not do

1. Never call the a^* axis red/green.
2. Don't assume that colors opposite one another are complementary colors.
3. Don't assume that two colors with the same CIELAB hue angle have the same perceptual hue.
4. Never, ever compute the color difference between two CIELAB values computed with different illuminants. Just don't do it.
5. Don't assume that CIELAB is perceptually linear.
6. Don't trust any color difference formula (even ΔE_{00}) when the illuminant is significantly different from D65.

Introduction

The genesis of CIELAB goes back to a paper by Eliot Adams in 1923 (Adams, 1923). In this paper, he proposed a simple mathematical model of the human visual system. The model included the three types of cones as the sensors and used a neural schematic to convert these into the perceptual signals that are described by Hering's color opponent theory. Although he did not formally state equations, the mathematical model was there.

Color measurement at this time was considerably different than today. Often "measurements" of color were performed with color matching device. The device would allow the operator to adjust the intensity of three light sources (generally red, green, and blue) in order to match the color to be measured. The settings of the three light sources would be used as a proxy for the measured value of the color. Since there were three stimuli used to measure color, the device was known as a called a *tristimulus colorimeter* (Seymour, 2020).

Such devices were difficult to use and had poor reproducibility. The next step in the development of CIELAB was the result of the efforts to create an improved version of the tristimulus colorimeter. It was realized at the time that a spectrophotometer could emulate tristimulus color measurements that were more repeatable and less subjective. The spectrophotometer would supply the spectral data and the math was developed to convert from spectral data to settings for the three light sources.

But since the constants in the math depended on the spectral characteristics of the light sources, these light sources needed to be standardized. The standardization became known as the 1931 Standard Observer. Since the committee was free to choose any light sources they wanted, the light sources were chosen so as to simplify the hand calculations. They are theoretical light sources, and are not even physically possible, since the light sources would need to generate a negative amount of light at certain wavelengths. Tristimulus colorimeters were replaced with spectrophotometers and arithmetic. Apart from the Standard Observer functions that have been dutifully tabulated in various standards for 90+ years, all that remains of the tristimulus colorimeter today is the odd phrase the *tristimulus* functions.

One important point for the purposes of this paper is that the 1931 Standard Observer does not represent an approximation of the spectral response of the human cones. A reasonable approximation of the cone functions can be made, however, from an appropriate linear combination of the three functions $\bar{x}$, $\bar{y}$, $\bar{z}$, that make up the Standard Observer.

Adams later revisited his model (Adams, 1942). In this paper he introduced a set of formulas that defined a color space, and he provided plots that showed a close correspondence between his new color space and the Munsell color space.

His equations were revised a bit with the help of Dorothy Nickerson. The final revision of the Adams-Nickerson color space was among a handful of color spaces that were considered by the CIE committee that developed CIELAB. Some further modifications were made and the revision was adopted as a standard in 1976.

If one looks just at Adams' 1923 paper, it would seem reasonable that Adams would build a color space on the spectral responses of the cones as he originally suggested. But rather, his 1942 color space was built on the 1931 Standard Observer. Adams did not foresee the consequences of this decision.

There is no indication in his 1942 paper that he gave the matter much thought. A thorough search of the papers written by people involved with 1976 CIE committee around 1976 (Wyszecki 1974, Hunter 1975, Judd and Wyczecki 1975, Kuehni 1976, McLaren 1976, Pauli, 1976, Lozano 1977, Schanda 1978, Billmeyer and Saltzman 1981, MacAdam 1981, and McLaren 1986) suggests that they, too, did not give much thought to the consequences of the decision to base CIELAB on the tristimulus functions.

The functions that we call the *tristimulus functions* would be more properly called "a set of functions that are used to convert spectral data so as to emulate a color matching device that hasn't seen much use since 1940." The functions $\bar{x}$, $\bar{y}$, $\bar{z}$, that we use to calculate CIELAB values do not exist in the eye, in the neural networks leading to the brain, or in the part of the brain used to decipher color. They only exist in tables, inside computer programs that deal with color, and in the brains of color scientists.

This paper revisits the tristimulus-friendly decisions made in 1931, 1942, and 1976, shining a light on the unforeseen consequences that limit CIELAB today.

The six foibles

The hue angle of red is 25°

Introductions to CIELAB generally refer to the a^* axis as the red-green axis. This can be seen, for example, on websites from many of the major spectrophotometer manufacturers (Datacolor, HunterLab 2008, Konica-Minolta 2018, Techkon, and XRite 2016).

Some of the literature goes further to explain that this is in line with the Hering theory of color opponents, which states that red versus green is one of the three sets of color opponents which form our intuitive understanding of color.

But red is about 25° counterclockwise from the a^* axis. This dates back to the original color space developed by Adams. It can be readily seen in Figure 1 which is taken from his 1942 paper, but went without mention. The figure shows that 5R

5/12 (a rich red) is 26° from the axis. (Note that the definition of CIELAB flipped the original Adams color space top-to-bottom.)

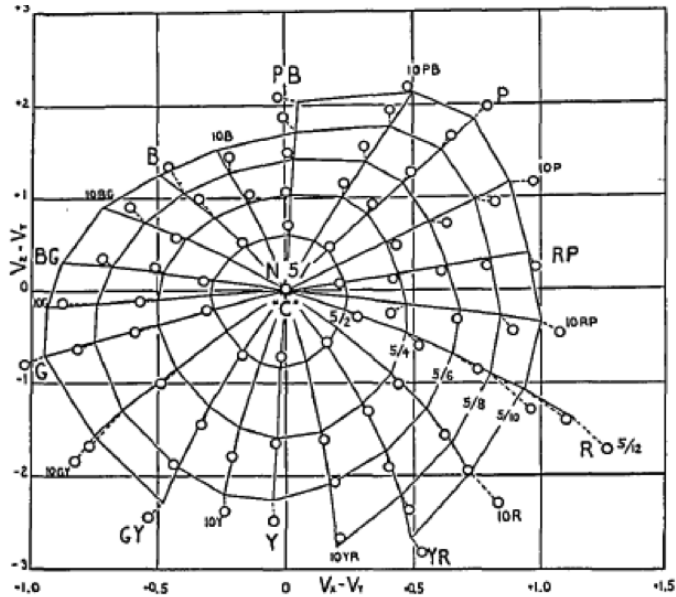


Figure 1 – Adams' chromatic value diagram of Munsell spiderweb at value of 5

In an earlier paper from this author (Seymour 2020), 5R samples from two Munsell books and -R samples from Natural Colour Systems were measured and converted to hue angle (Figures 2 and 3).

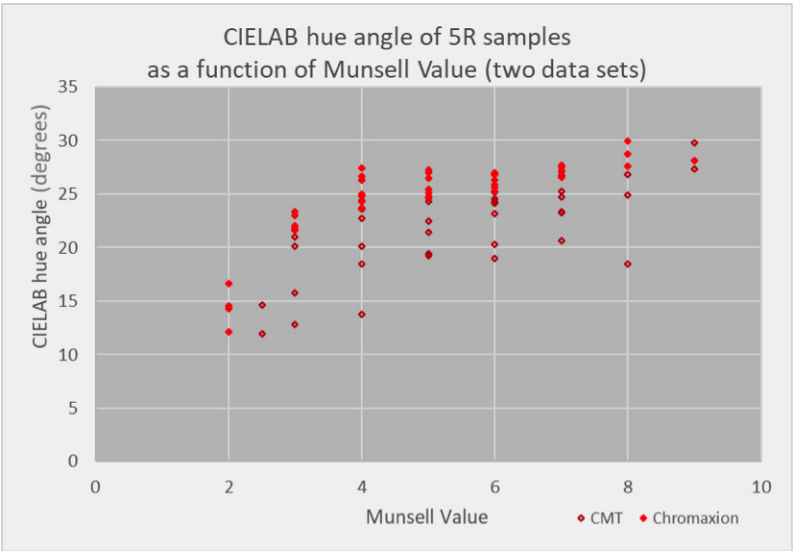


Figure 2 – Two versions of Munsell 5R red

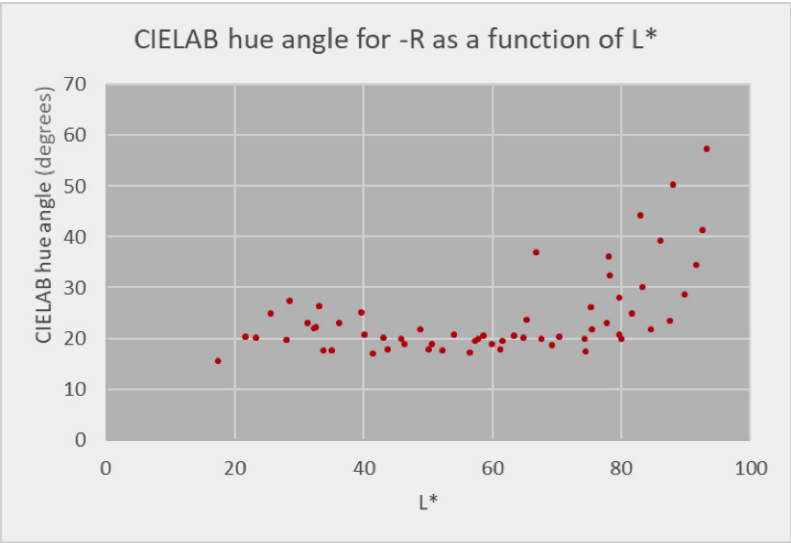


Figure 3 – Hue angle of pure red NCS samples, as a function of L^*

The paper provides additional examples, but the consensus is that red has a CIELAB hue angle of between 20° and 30°. In the paper, Seymour demonstrates that this rotation is an unintended consequence of the use of the Standard Observer. If Adams had stayed with his original concept of using the cone functions, his positive horizontal axis would have pointed substantially in the red direction.

This is clearly not a big practical issue. Aside from some initial confusion, the rotation of the color space is not a significant issue. But it might be nice to have a color space that puts the key colors in the correct locations.

Complementary colors are not always directly across from each other in CIELAB

In the early 18th century, Jakob Christof LeBlon began experimenting with color printing. Originally inspired by Newton’s naming of the colors of the rainbow, he started with seven colors of transparent printing inks. By the time he was granted a patent in 1719, he had reduced the set to three inks: red, yellow, and blue. From these, he asserted, you could create all colors. (Hardy and Wurzburg 1937, Shevell 2003).

These colors are known today as the artists’ primary colors. By combining them in pairs, you could get the artists’ secondaries: orange, green, and purple. The first color wheel was printed in 1766 by Moses Harris. The contemporary 12 step version of the color wheel was developed by the French chemist and dyer Michel Chevreul. His color wheel is depicted on the left in Figure 4. From this we can see various complementary pairs, including red-green, orange-blue, and violet-yellow. The table below shows some basic color names and their CIELAB hue angles {(adapted from Seymour, 2016). The 12 step CIELAB color wheel can be depicted on the right as in Figure 4, based on this table.

Color	Hue angle
Magenta	0°
Red	30°
Orange	60°
Yellow	90°
Green	145°
Cyan	225°
Blue	270°
Purple	315°

Table 1 – Hue angles for some basic colors

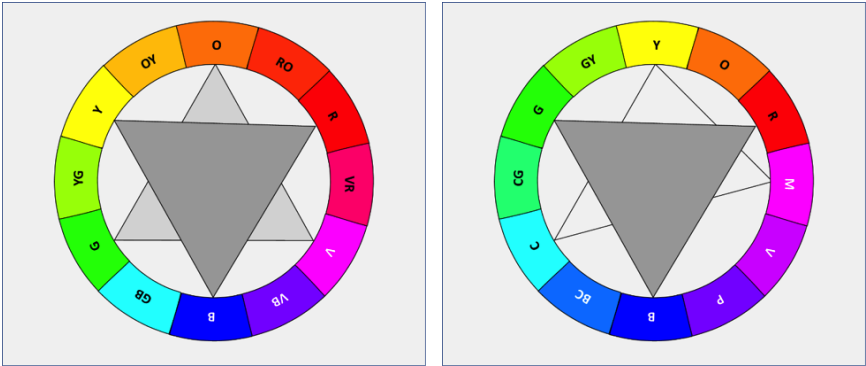


Figure 4 – Artist's color wheel (left) and CIELAB color wheel (right)

The two color wheels are arranged so that red is in the same position and have orange counterclockwise from that. It can be seen that the angular distance between red and yellow is considerably larger for the artists' color wheel, and the angular distance between green and blue is considerably larger for the CIELAB wheel.

A third color wheel can be derived from Munsell’s color system. In Munsell’s original system, the complementary colors were assigned so that an additive light mixture of the colors created gray. One could argue that this is the most physiologically based system, since it starts with basic principle that complementary colors make gray when additively mixed in the right proportions.

Color	Artists' complement	CIELAB complement	Munsell complement
Red	Green	Cyan	Blue-Green
Orange	Blue	Blue-Cyan	Blue
Yellow	Purple	Blue	Purple-Blue
Green	Red	Violet	Red-Purple
Cyan	Red-Orange	Red	Red-Orange
Blue	Orange	Yellow	Orange
Purple	Orange-Yellow	Green-Yellow	Yellow-Green
Magenta	Yellow	Cyan-Green	Blue-Green

Table 2 – Comparison of color complements based on different systems

It is seen from the table that there is not full agreement on complementary colors. Part of this may be due to the ambiguity of color names, and some may be due to the ambiguity of the phrase *complementary colors*. Regardless, don't assume that CIELAB will predict complementary colors that agree with other systems.

Hue lines in CIELAB are warped

In the original Munsell color system, Munsell assigned hue and chroma based on a Maxwell disk. In this device, colors of two samples were mixed by spinning the disk fast enough to blur them together. Munsell began with color of five paints for his five primary colors. Five more colors – secondary colors – were created by finding paints that when spun on the Maxwell disk would create gray. These complementary colors filled out the ten major hues on the Munsell color circle. Presumably, additional gradations of hue were filled in by suitable mixtures of adjacent primaries.

In 1935, various color scientist of the day became interested in seeing the perceptual linearity of the Munsell Color system improved. In 1937, the Optical Society of America appointed a subcommittee with the intent of improving the perceptual linearity of the Munsell color system. Three million observations were made by forty observers to assess the linearity, and Munsell notations were adjusted accordingly. The results were then subjected to smoothing to generate the final data set known as the Munsell Renotation data (Newhall et al. 1943).

Figure 5 is a slice of the Munsell color space as defined by the Munsell renotation data. These are the data points in the color space which has a Munsell Value of 5, which corresponds to $L^* = 50$ in CIELAB. Each of the curved lines are lines of constant hue according to the Munsell renotation data.

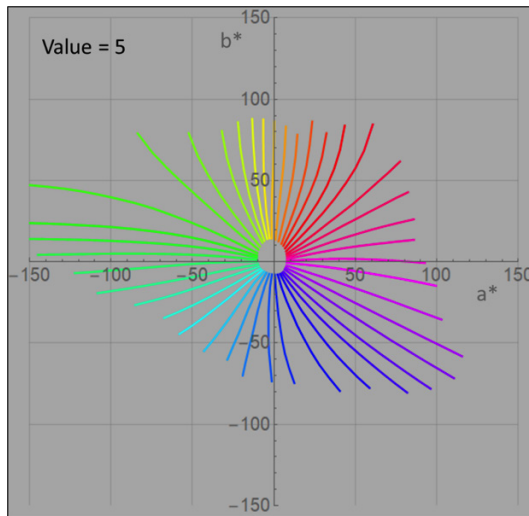


Figure 5 – Spidergram showing hue differences between CIELAB and Munsell at a Munsell Value of 5. If CIELAB and the Munsell renotation data agreed on hue assignment, these would be straight lines. This is true for certain hues, but there is significant curvature around a hue angle of 60°, on either side of 135°, and around 270°.

The two color systems disagree in many cases about whether pairs of colors have the same hue, but which one is “correct”, that is, which one agrees better with human perception? Fortunately, there is another data set to compare against CIELAB hue. The Swedish Natural Colour System (NCS) is another three-dimensional color system, which like Munsell Color system, includes a physical atlas of roughly 2,000 colors. It was developed independently of any work with the Munsell system, with the first version of the NCS atlas being introduced in 1966 (Hård 1966).

The method by which hues were assigned in the Swedish Natural Colour System (NCS) has been carefully explained (Hård et al. 1996). The hue experiment began with 37 subjects being asked to select the four chromatic Hering colors: red, yellow, green, and blue. They were asked (for example) to identify the color that is red, without a trace of blue or yellow. There was very good agreement among the subjects.

In the next experiment, they filled out the hue circle. The subjects were given a high chroma reference color and another color of the same lightness and chroma but with slightly different hue. They were asked to find a color that had the same perceptual hue difference as the two, but was on the other side. A sequence of these judgments was used to create a perceptually linear hue circle. Finally, for each of the samples on the hue circle, the subjects were asked to identify, from among 30,000 samples, those that were of the same hue as the reference.

From this explanation, it is clear that the assignment of hue in NCS was careful, deliberate, and based on human judgement of color. All colors that have been assigned the same NCS hue designation have been vetted by human observers.

Further, the data used to create NCS was completely disjoint from that used to generate the Munsell renotation data. If the differences between CIELAB and NCS are similar to those between CIELAB and Munsell Renotation data, then those differences are conclusively errors in CIELAB.

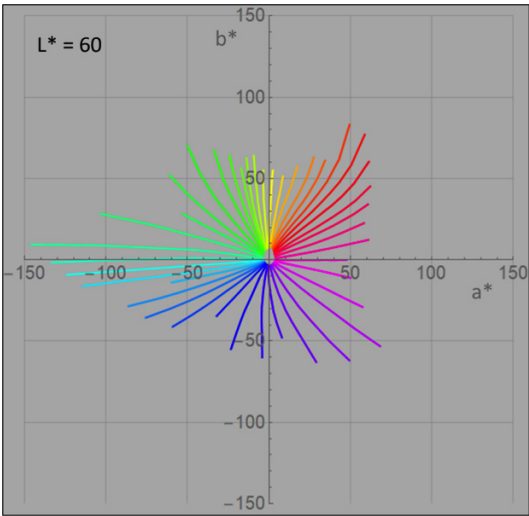


Figure 6 – Spidergram showing hue differences between CIELAB and NCS at an L^* near 60

Figure 6 shows a similar plot of NCS data near $L^* = 60$. Overall, the warping of the lines of NCS constant hues is very similar to that of the Munsell constant hues. If two colors have the same CIELAB hue angle it does not follow that they have the same perceptual hue. Using CIELAB to create a harmonious palette with monochromatic hue will not always be successful, despite that fact that palettes with constant perceptual hue are perhaps the most universally accepted as being harmonious.

Another issue with CIELAB hue inconstancy in the graphic arts has to do with gamut compression. When a blue/purple color is decreased in chroma to fit a device with a smaller gamut, a significant hue shift has been noted. It was demonstrated that this deleterious effect can be attributed to the unfortunate choice of XYZ values to compute CIELAB (Moroney 2000).

D50/2 and D65/2 are different color spaces

CIELAB values are routinely computed under different illuminants for the purpose of determining metamerism. Two printed samples may be rendered with different sets of pigment, either by printing on devices with different process colors or by printing with spot colors mixed using different base pigments. When a pair of colors are an acceptable match under one illuminant, they are often checked using CIELAB values computed under a second illuminant. In this case, color difference formulas are being used appropriately. Color differences are computed between the two samples under D50/2 (for example), and then again between the two colors under a second illuminant. Color differences in both cases are computed between pairs of colors under the same illuminant.

It is tempting to compare the CIELAB values of a single sample computed under two different illuminants e.g., D50/2 and D65/2. Presumably, this would be an indication of the color inconstancy, that is how much perceptual change there is due to a change in illuminant. This would be a potentially useful metric, but CIELAB (and all color difference formulas based on it, including ΔE_{00}) would not be a good way to compute that.

The diagram below (Seymour 2022) compares the change in color values of seven sets of D50/2 metamers in the yellow/green quadrant when the illuminant changes to D65/2. On the left side are the changes in CIELAB values. The much smaller color changes are shown at the right when a different color space, Conelab2, is used to compute the color coordinates. The two color spaces differ in that CIELAB is computed from XYZ values, whereas ConeLab2 is computed from LMS values, which represent the spectral response of the eye. ConeLab2 is a much closer emulation of what happens in the human visual system when the illuminant is changed.

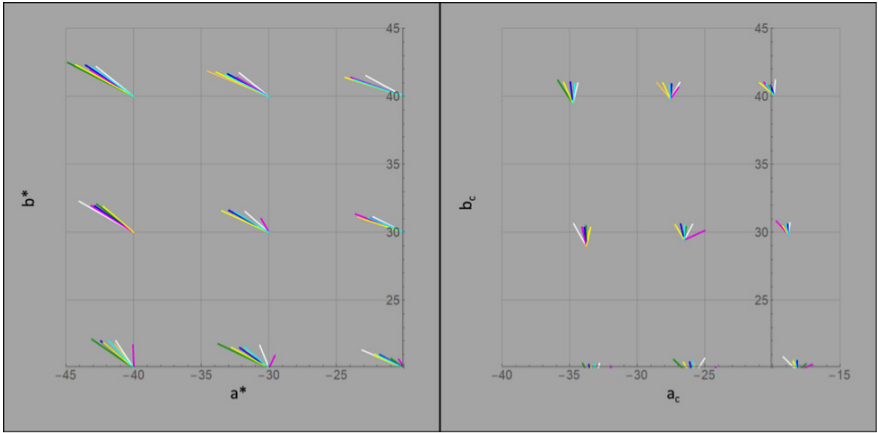


Figure 7 – Closeup of upper left-hand corner of Figure 12

The color shifts due to the illuminant change predicted by CIELAB are an average of 2.3 times as large as those predicted by ConeLab2. The conclusions from this are first, that color changes due to illuminant change are poorly predicted by CIELAB, and that the culprit is that CIELAB does normalization in XYZ values rather than LMS values.

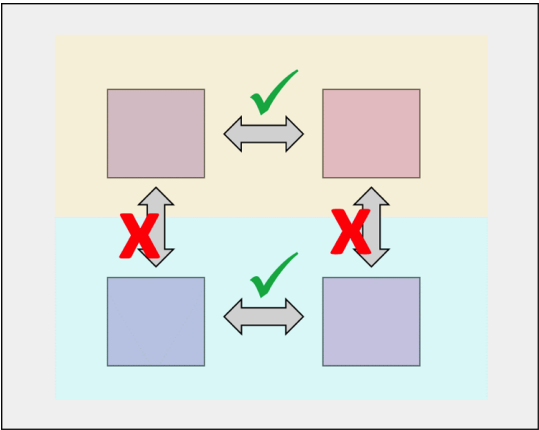


Figure 8 – Permissible color difference calculations

D50/2 and D65/2 are different color spaces (Brill, 2021). Color differences should not be calculated between different color spaces using CIELAB. This could be a meaningful comparison in a color space based on LMS.

CIELAB is far from perceptually linear

The big impetus for developing CIELAB was for determining if a color is in production tolerance. The intent was for color differences computed with CIELAB to correlate well with color acceptability assessments made by industry experts, with 1 ΔE being a tight tolerance applicable for all colors.

The deviation from this intent was noted in ANLAB (the predecessor to CIELAB) by McLaren in 1971 (McLaren 1971). He noticed a pattern to the discrepancies and, rather than fix ANLAB, he applied a kludge to the difference values that made a better fit to his pass/fail data. This is like selling longer rulers to folks in Greenland because the Mercator projection makes their county look huge. This idea of changing the formula for ΔE rather than changing the original color space found its way into ΔE_{JPC79} (based on CIELAB), ΔE_{CMC} , ΔE_{94} , and finally to the current standard ΔE_{00} (as well as a few more).

This is unfortunate. Ideally, we would be using a color space which is closer to being perceptually linear. Discussion of which color space (currently existing or new) would meet this need is beyond the scope of this paper, but two areas for improvement are provided.

First, the nonlinearity function f in the CIELAB computations is not ideal, but rather a compromise that got the equation through committee. An alternative nonlinear equation has been suggested (Seymour 2015). The new equation provides one axis of a color space that agrees favorably with the ΔE_{00} equation for the L^* contribution to color difference. In addition, this equation is considerably simpler than the computations involved in ΔE_{00} ; in fact, it is simpler even than the equation for L^* . The simplicity of the formula suggests that adjusting the constants in this formula could adapt it for different absolute light levels.

$$L_{00} = 24.7 \log_e(20Y+1) \tag{1}$$

The graph below shows the discrepancy between color differences along the neutral axis, as computed using the L_{00} equation versus the ΔE_{00} equations.

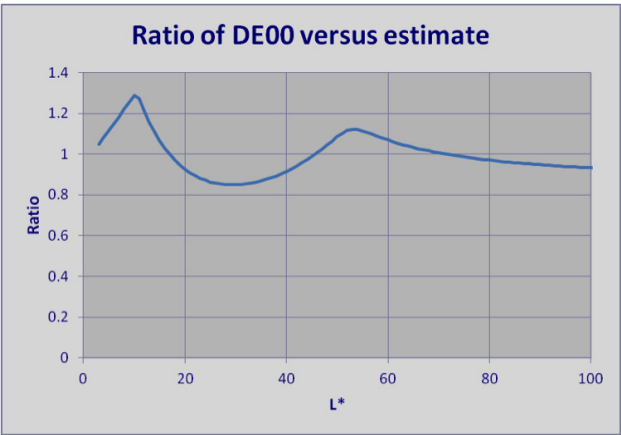


Figure 9 – Comparison of the two formulas for color difference

So, one improvement to CIELAB would be to utilize a different function to emulate the nonlinearity of the human visual system. Equation 1 is one candidate.

A second improvement to CIELAB would be to use LMS in place of XYZ as input to the equations. As discussed before, there are several reasons for this substitution that do not involve perceptual linearity. The L, M, and S signals actually exist in the human visual system whereas X, Y, and Z do not. It seems plausible that a color space which more closely emulates the human visual system would have a better chance to emulate its nonlinearity.

Figure 10 is one demonstration that LMS could improve perceptual linearity. Visual assessments of color differences showed an anomaly in the blue regions. Tolerance regions are normally oriented such that their major axes are along constant hue lines. However, these regions are rotated counterclockwise in the blue region. To compensate for this, a correction was added to the ΔE_{00} equations in the vicinity of

the 275° hue angle (CIE 2001). The resulting tolerance ovoids are illustrated on the left in Figure 10.

The graph at the right in Figure 10 is a section from Figure 5, showing the Munsell constant hue lines. In the blue region, the rotation is similar to the rotation in ΔE_{00} . This suggests the possibility that a color space that more closely followed the constant hue lines of Munsell space might not need the blue rotation in ΔE_{00} . Presumably a color space based on LMS would avoid the need for this rotation correction.

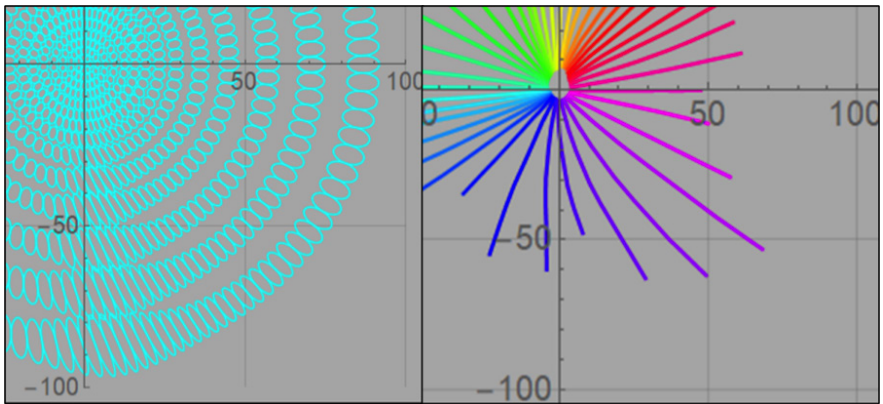


Figure 10 – Comparison of ΔE_{00} tolerance ellipses (left) with Munsell constant hue curves (right)

From a practical standpoint, this might not seem that important. Perhaps a more elegant set of formulas are possible, but what we currently have works well enough for people to not complain. On the other hand, if the need for the blue ovoid rotation and the blue hue anomaly of Munsell are due to the same cause, then it seems likely that the other problematic hue angles from Figure 5 might also require an ovoid rotation. It may be that ΔE_{00} would benefit from additional ovoid rotation corrections around a hue angle of 60°, and opposing rotations on either side of 135°.

Or, rather than add a few more pails to catch the drips, it might be time to consider fixing the leaky pipe. It appears that a judicious choice of nonlinearity function and the use of LMS could lead to a simple set of equations that yield a color space which is perceptually linear enough to not require a color difference equation to make up for residual nonlinearity.

Color difference under illuminants other than D65 and Illuminant C

There has been a great deal of work done creating data sets to be used for validation of color difference formulas. The original work in this direction was done under the now-obsolete Illuminant C. Almost all the more recent work has been done under D65 with the 10° observer.

If CIELAB basically works well, then it would be a reasonable guess that the ΔE_{00} color difference equation would work fairly well for D50, which is to say, it would

correlate well with human perceptual judgements of color differences under D50. From the experience in the industry, this seems to be the case.

But it should not be automatically assumed that ΔE_{00} will work well under illuminants that differ significantly from D65. Figure 11 illustrates the distortion of color space when going from a rectangular grid under D50/2 to A/2. (At each grid point location, the spectra for seven metamers under D50/2 were converted to $L^*a^*b^*$ under A/2. These seven color coordinates were then averaged.)

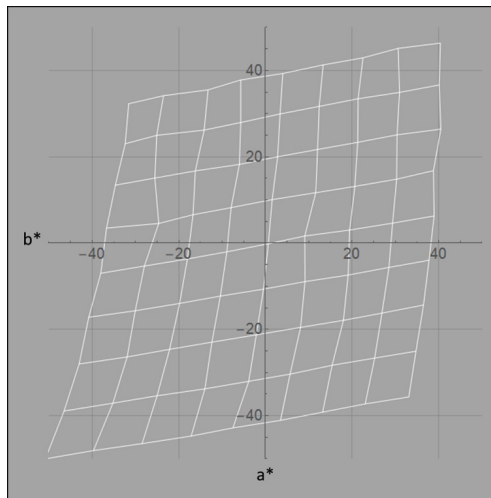


Figure 11 – Distortion of CIELAB values from D50/2 to A/2 at $L^* = 50$

The systematic distortion is readily apparent. The grid is compressed along one diagonal and expanded along the other. A pair of rich yellow-green samples (from the upper left-hand quadrant) that differ by 2 points in C^* under D50/2 may differ by only 1 point under A/2. On the other hand, a pair of cyan samples (lower left-hand quadrant) that originally differ in C^* by 2 points might be 3 or 4 points apart.

It is perfectly reasonable that a large change in the color temperature of the illuminant would tend to enhance some color differences and minimize others, but is this real? I would argue that the change we see is suspect. First, one would expect that warm light, being deficient at the blue end, would decrease discrimination in b^* . But examining Figure 11, there is very little change in C^* for colors along the b^* axis.

Second, as demonstrated in a previous section (“D50/2 and D65/2 are different color spaces”), when the illuminant is changed from D50 to D65, a large portion of the change in color coordinates is due to the unfortunate decision to normalize based on XYZ rather than based on LMS.

Thus, it is reasonable to question whether color difference equations faithfully represent our perception of color difference when the illuminant is much different from D65.

Summary

This has been a tour of the dark and dusty closets of CIELAB. This industry standard color space has been shown to not work well for six tasks which are seemingly within its job description. Some of these are trivial, but others are of some potential consequence. Most of the foibles and fumbles can be traced back to the unfortunate choice of basing the computations on XYZ instead of LMS.

It is hoped that this paper encourages the search for a CIEALB-like color space which is based on LMS and which does not suffer from the irregularities of CIELAB.

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