Understanding Fluorescence – Measurement, Simulation and Correction

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Abstract

Optical brighteners are often used to make yellowish paper or media appear whiter and brighter, resulting in a product that is much more appealing. The operative aspect of optical brighteners is that they use fluorescence to convert ultraviolet light (which is invisible to human vision) to blue light (which is visible). The amount of fluorescence is related to the light in the ultraviolet portion of the spectrum and changes under different light sources. However, this poses problems for establishing color management profiles using a spectrophotometer to measure spectral reflectance to derive colorimetry of printed colors under arbitrary light sources on media. The challenge is that the derived colorimetry doesn't match the actual color appearance, resulting in undesirable color casts in the final printed output using color management.

In this paper, a comparison of measuring spectral reflectance versus measuring spectral fluorescence will be reviewed along with the mathematical differences related to spectral reflectance and spectral fluorescence. It is shown that colorimetric measurements derived from spectral reflectances of surfaces that have fluorescence are only valid for the light source used in the spectrophotometer to measure the spectral reflectances. This is because the application of a light source to spectral reflectance to predict the light reflected off the surface doesn't consider the hidden fluorescence when the spectral reflectance measurement was made.

A method of simulating spectral fluorescent measurements is proposed based on an extension of the adjustment used in Substrate Colorimetric Correction Aims (SCCA) by first applying the approach to spectral reflectance and then spectral fluorescence. A simple physical model related to fluorescent substrate correction is presented. Having the ability to use simulated measurements eliminates the need to use expensive measurement equipment to get spectral fluorescent measurements for the purposes of evaluating the extent of divergence and error when using spectral reflectance measurements of colors on media with optical brighteners.

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A set of spectral fluorescent measurements were then estimated using this proposed method of deriving spectral fluorescent measurements as part of a virtual experiment for colors of an IT8.7/4 chart on a hypothetical media with an optical brightener with significant fluorescence. Corresponding spectral reflectance measurements were then derived from this set using a virtual spectrophotometer with various embedded light sources. This enabled comparisons to be made between the resulting spectral reflectances and their predicted colorimetry to gain insight into the extent of colorimetric shifting between applying different light sources/illuminants to reflectances having fluorescence (measured with different embedded light sources). Reporting, observations, and recommendations from this virtual experiment are then made.
One conclusion from this virtual experiment is that colorimetry for spectral reflectances (that involve fluorescence) under a desired light source/illuminant is only correct for the light source that was used in the spectrophotometer to make the measurement. Repurposing spectral reflectances to other light sources/illuminants results in errors in predicting the color appearance.

One question that resulted from this experiment was whether some form of correction could be applied to spectral reflectance measurements to better predict the colorimetry (and actual color appearance) for a different light source than the one that was used to make the measurement. An approach was tried using the set of estimated spectral fluorescent measurements to derive such a correction. This correction doesn't exactly predict the colorimetric results using spectral fluorescent measurements. However, it does provide improved prediction with less colorimetric error. There is one challenge with this approach in that it requires a correct understanding of the fluorescent behavior of the media.

Future research efforts could involve finding correlations with measurements to spectral fluorescent media modeling to allow for corrections to be applied to actual spectral reflectance measurements to get better prediction of color appearance with fluorescence under arbitrary light sources.

Introduction

A wavelength of light can interact with an object's surface in several ways. It can be absorbed. The light can bounce off the surface like a mirror (surface reflectance). The light can enter the surface and bounce around before exiting the opposite surface (known as transmittance). And lastly, the light can enter the surface, get absorbed (adding extra energy to molecules in the surface), and then this energy is released as emitted light at a longer wavelength (at a lower energy level) exiting the surface. In this last case, fluorescence occurs when the wavelengths of excitation light onto a surface are not the same as the wavelengths of light emitted from the surface.

One practical use of fluorescence is in the use of optical brighteners that are added to make yellowish paper or media appear whiter and brighter - resulting in printed output that is much more appealing. The operative aspect of optical brighteners is that they use fluorescence to convert ultraviolet light (which is invisible to human vision) to blue light (which is visible). Inks can also fluoresce resulting in brighter, more chromatic output. In such cases the surface appears to glow due to more light being emitted at wavelengths from the surface than are available in the viewing illumination.

Fluorescence poses a challenge to the practice of color management because the measurement of colors used to characterize and profile the output doesn't adequately capture how the surface color's appearance changes as the viewing lighting conditions change. This is because the measurements and related mathematical representation of color do not account for fluorescence.

In this paper we will explore how color is typically measured (as reflectance), how this fails to account for fluorescence, and how fluorescence could better be measured. Parallel to this is a review of both the mathematical model of color that corresponds to reflectance-based color measurement and how this relates to a mathematical model for fluorescence. A model of estimating fluorescence is presented allowing for a quantitative comparison and analysis between using reflectance-based color measurement and fluorescence-based color measurement. Lastly, the possibility of correcting reflectance measurements to better predict fluorescent appearance is explored.

Measurement

The basic elements of measuring spectral reflectance using a spectrophotometer are outlined in Figure 1. In this figure, a source of light is used to illuminate a surface that is being measured. The reflected light is separated into its constituent wavelengths and a spatial light sensor is used to measure the amount of light reflected at each wavelength. (Note: measurement geometry includes the angles of the light source and measurement sensor as well as whether and how specular reflectance is part of the measured light).

Calibration of the system is performed by taking a measurement of the light source of a perfectly diffusing white surface. The relative amount of light reflected at any wavelength (λ) of a measured surface (r_λ) can be expressed as the ratio of measured light of the surface (e_λ) divided by the measured light of the white calibration surface (w_λ).

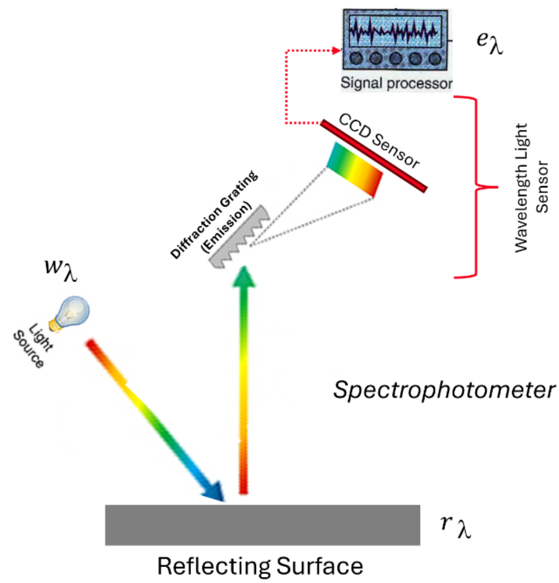

Figure 1 – Measuring spectral reflectance

This is known as reflectance factor and is expressed by Equation (1) for a single wavelength of light. The spectral reflectance of a surface over a range of *m* wavelengths is generally expressed as a vector (**r**) of reflectance factors as shown in Equation (2).

$$r_\lambda = \frac{e_\lambda}{w_\lambda} \tag{1}$$

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_m \end{bmatrix} \text{ for } \lambda\text{=1 to } m(\text{w}) \tag{2}$$

One thing that should be noted is that the measurement of reflectance factor has the built-in assumption that the wavelengths of light emitted from the surface are the same as those in the light source.

In contrast to this, the basic elements of measuring spectral fluorescence using a spectrofluorimeter are outlined in Figure 2. In this figure, the constituent wavelengths of a source of light are separated and a narrow-wavelength band of light is selected to illuminate the surface that is being measured.

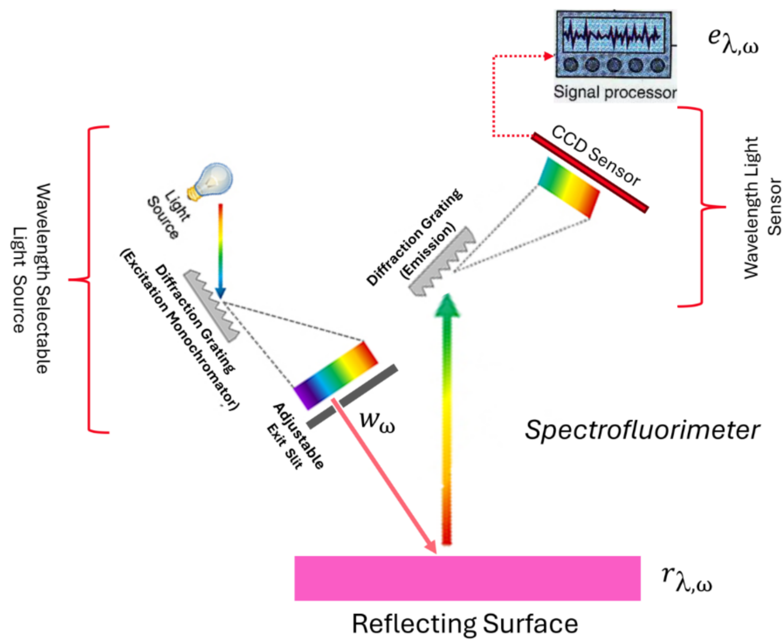

Figure 2 – Measuring spectral fluorescence

The reflected light is then separated into its constituent wavelengths and a spatial light sensor is used to measure the amount of light reflected at each wavelength. This results in a reflectance vector measured for the selected wavelength (ω) of the source light as shown in Equations 3 and 4.

$$r_{\lambda,\omega} = \frac{e_{\lambda,\omega}}{w_\omega} \tag{3}$$

$$\mathbf{r}_\omega = \begin{bmatrix} r_{1,\omega} \\ \vdots \\ r_{m,\omega} \end{bmatrix} \text{ for } \lambda\text{=1 to } m \tag{4}$$

This process is repeated by changing the selection of the source wavelength to measure a reflectance vector for each wavelength (ω) of the light source. A measurement of fluorescence represents the combination of the reflectance vectors for wavelengths of the light source and is represented as a matrix (**F**) as shown in Equation 5.

$$\mathbf{F} = [\mathbf{r}_1 \quad \cdots \quad \mathbf{r}_n] \quad \text{for } \omega\text{=1 to n} \tag{5}$$

This is commonly referred to as a Donaldson matrix.[1]

Note: The range of wavelengths in the light source is not generally the same as the reflected wavelengths of light, and calibration is similar to the reflectance measurement system but much more involved.

An example of a fluorescence measurement of a media with an optical brightener is shown in Figure 3 with the corresponding Donaldson matrix in Figure 4. This measurement was performed by Brian Gamm while at the Munsell Color Science Laboratory using a Murakami Spectrofluorimeter.

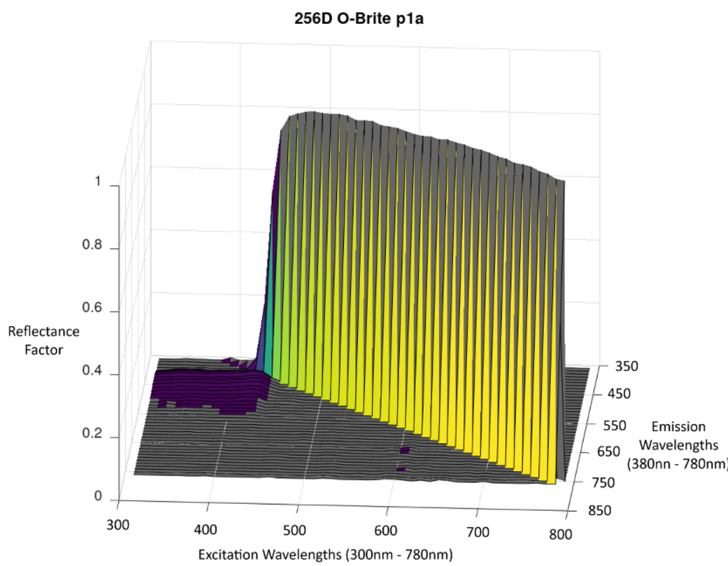

Figure 3 – Spectrofluorescent measurement of a media with an optical brightener (courtesy of Brian Gamm)

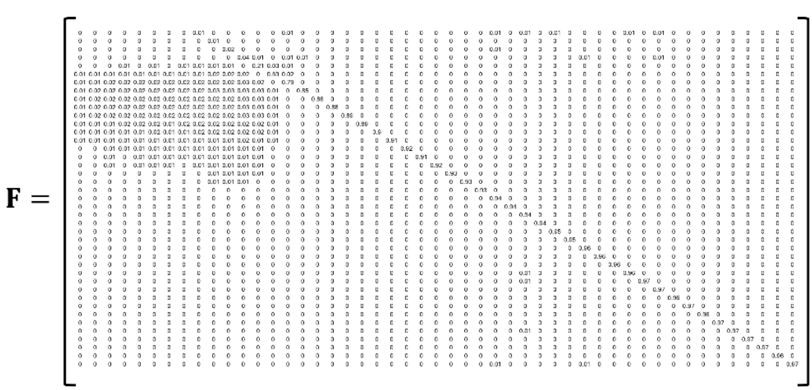

Figure 4 – Corresponding Donaldson matrix to Figure 3

As can be seen from this figure, the spectral reflectance portion forms a diagonal in the matrix where the excitation wavelengths are the same as the emission wavelengths. The off-diagonal portion in the upper left-hand portion of the matrix provides a measure of how much the optical brightener converts ultraviolet light into visible light at different wavelengths. Note: other off-diagonal matrix entries represent measurement noise and the challenges of calibration.

Emitted Light and Colorimetry

The amount of light emitted (e_λ) for a given wavelength (λ) due to reflectance from a surface can be expressed by rearranging Equation 1 as the product of the reflectance factor (r_λ) and the spectral power of the light source (w_λ) as shown in Equation 6. Equivalent vector representations are shown in equation 7 where the emittance vector ($\mathbf{e}_r$) is the scalar product of the reflectance ($\mathbf{r}$) and light source ($\mathbf{w}$) vectors, or in Equation(s) 8 where the emittance vector is the product of one of the reflectance or light vectors as a diagonal matrix multiplied by the other vector.

$$e_\lambda = r_\lambda w_\lambda \quad (6)$$

$$\mathbf{e}_r = \mathbf{r} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{r} \quad (7)$$

$$\mathbf{e}_r = \begin{bmatrix} r_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_\mathrm{n} \end{bmatrix} \mathbf{w} = \begin{bmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_\mathrm{n} \end{bmatrix} \mathbf{r} \quad \text{for } \lambda = 1 \text{ to n} \quad (8a)$$

$$\mathbf{e}_r = diag(\mathbf{w})\mathbf{r} \quad (8b)$$
$$\mathbf{e}_r = diag(\mathbf{r})\mathbf{w} \quad (8c)$$

The amount of light emitted from a surface that fluoresces ($\mathbf{e}_f$) is expressed as the product of the fluorescent Donaldson matrix ($\mathbf{F}$) by the light source vector ($\mathbf{w}$) in Equation 9.

$$\mathbf{e}_f = \mathbf{Fw} \quad (9)$$

(Note: Equation 8c is equivalent to Equation 9 with no off-diagonal terms).

A measurement of colorimetry of the light emitted from the surface is often represented as a normalized integral of the products of the color matching function, reflectance and light for each wavelength. This can also be represented as a matrix equation as shown in Equation 10.

$$\mathbf{c} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{Ce} \tag{10}$$

Where:

$$\mathbf{C} = \frac{100}{k}\begin{bmatrix} \bar{\mathbf{x}}^T \\ \bar{\mathbf{y}}^T \\ \bar{\mathbf{z}}^T \end{bmatrix},$$

$\bar{\mathbf{x}}^T, \bar{\mathbf{y}}^{T,}, \bar{\mathbf{z}}^T$ represent transposed color matching function vectors, and

$$k = \bar{\mathbf{y}}^T \boldsymbol{w}$$

From Equation 10, the measured colorimetry based on reflectance ($\mathbf{c}_{\boldsymbol{r}}$) is expressed in Equation 11 and the measured colorimetry based on fluorescence ($\mathbf{c}_{\boldsymbol{f}}$) is expressed in Equation 12.

$$\mathbf{c}_{\boldsymbol{r}} = \begin{bmatrix} X_r \\ Y_r \\ Z_r \end{bmatrix} = \mathbf{Ce}_{\boldsymbol{r}} \tag{11}$$

$$\mathbf{c}_{\boldsymbol{f}} = \begin{bmatrix} X_f \\ Y_f \\ Z_f \end{bmatrix} = \mathbf{Ce}_{\boldsymbol{f}} \tag{12}$$

Measuring fluorescence as reflectance

The measured reflectance of a surface can be expressed in a vector form of Equation 1 as follows in Equation 13:

$$\mathbf{r} = \begin{bmatrix} \frac{\mathbf{1}}{\boldsymbol{w_1}} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{\mathbf{1}}{\boldsymbol{w_n}} \end{bmatrix} \boldsymbol{e} = diag(\boldsymbol{w})^{-\mathbf{1}}\boldsymbol{e} \tag{13}$$

This can be re-expressed to show the calculated reflectance ($\mathbf{r}_{\boldsymbol{r}}$) relative to the measurement light source ($\mathbf{w}_{meas}$) given the measured emitted light due to reflectance ($\mathbf{e}_{\boldsymbol{r}}$) as given by Equation 14.

$$\mathbf{r}_{\boldsymbol{r}} = diag(\mathbf{w}_{meas})^{-\mathbf{1}}\mathbf{e}_{\boldsymbol{r}} \tag{14}$$

Substituting ($\mathbf{e}_{\boldsymbol{r}}$) from Equation 8b into Equation 14 and simplifying shows that the calculated reflectance is independent of the measurement light source as follows:

$$\mathbf{r}_r = diag(\mathbf{w}_{meas})^{-\mathbf{1}} diag(\boldsymbol{w}_{meas})\boldsymbol{r} = \boldsymbol{I} \cdot \boldsymbol{r} = \boldsymbol{r} \tag{15}$$

Similarly, the calculated reflectance ($\mathbf{r}_f$) relative to the measurement light source ($\mathbf{w}_{meas}$) with the measured emitted light due to fluorescence ($\mathbf{e}_f$) is given by Equation 16.

$$\mathbf{r}_f = diag(\mathbf{w}_{meas})^{-\mathbf{1}}\mathbf{e}_f \tag{16}$$

Substituting ($\mathbf{e}_f$) from Equation 9 into Equation 16 results in an equation that cannot be further simplified (see Equation 17).

$$\mathbf{r}_f = diag(\mathbf{w}_{meas})^{-\mathbf{1}}\boldsymbol{F}\,\boldsymbol{w}_{meas} \tag{17}$$

Thus, the calculated reflectance when fluorescence occurs is dependent on the measurement light source. This is an important fact when using reflectance to determine colorimetry when fluorescence is present. This is because colorimetry is based on the predicted emission from a surface having a calculated reflectance under an alternate light source. This is expressed using Equation 8b to predict the emission ($\boldsymbol{e}_{r,alt}$) from a surface having a given reflectance ($\boldsymbol{r}$) for an alternate light source ($\boldsymbol{w}_{alt}$) as follows:

$$\boldsymbol{e}_{r,alt} = diag(\boldsymbol{w}_{alt})\boldsymbol{r} \tag{18}$$

Substituting Equation 17 in for ($\boldsymbol{r}$) in Equation 18 results in the following:

$$\boldsymbol{e}_{f,alt} = diag(\boldsymbol{w}_{alt})\boldsymbol{diag}(\boldsymbol{w}_{meas})^{-\mathbf{1}}\boldsymbol{F}\boldsymbol{w}_{meas} \tag{19}$$

However, Equation 19 can only be shown to match the actual emission when fluorescence occurs (as expressed in Equation 9) when the alternate light source is the same as the measurement light source. Thus:

$$\begin{gathered}\boldsymbol{w}_{alt} = \boldsymbol{w}_{meas}\\ \boldsymbol{e}_{f,meas} = diag(\boldsymbol{w}_{meas})\boldsymbol{diag}(\boldsymbol{w}_{meas})^{-\mathbf{1}}\boldsymbol{F}\boldsymbol{w}_{meas}\\ \boldsymbol{e}_{f,meas} = \boldsymbol{I}\boldsymbol{F}\boldsymbol{w}_{meas}\\ \boldsymbol{e}_{f,meas} = \boldsymbol{F}\boldsymbol{w}_{meas}\end{gathered} \tag{20}$$

The take home message from this is that colorimetry from reflectance measurements using a spectrophotometer is ONLY CORRECT when the illuminant / light source used to determine the colorimetry is the SAME AS the light source in the spectrophotometer used to make the measurements. When the illuminant/light source used to determine colorimetry isn't the same as the "measurement" light source then the colorimetry WILL NOT MATCH the experienced appearance.

A follow-up question to this conclusion is to ask, "How bad a problem is using measured reflectance measurements when fluorescence occurs with arbitrary illuminants/light sources to predict colorimetry?" This requires having fluorescent measurements for comparison purposes, but measuring fluorescence requires an apparatus that is likely not available or easy to set up. For quantification purposes we will consider an approach to estimate spectrofluorescence measurements based on reflectance measurements of printed output and a spectrofluorescence measurement of a media.

Fluorescent Measurement Simulation

The process of adjusting measurements from one medium to another is commonly performed using the SCCA substrate correction algorithm[2] which is functionally equivalent to media-relative adjustment used by ICC color management.[3] The essence of substrate correction is a white point normalization that divides the tristimulus values of the source measurement by the tristimulus values of the source medium and then multiplying by the tristimulus values of the alternate medium. In equation form this looks like equation 21. The normalization matrix can be split into two diagonal matrices (equation 22) which can then be expressed as a diagonal matrix operators applied to the source colorimetric vector (equation 23).

$$\begin{bmatrix} X_{alt} \\ Y_{alt} \\ Z_{alt} \end{bmatrix} = \begin{bmatrix} X_{w_{alt}}/X_{w_{src}} & 0 & 0 \\ 0 & Y_{w_{alt}}/Y_{w_{src}} & 0 \\ 0 & 0 & Z_{w_{alt}}/Z_{w_{src}} \end{bmatrix} \begin{bmatrix} X_{src} \\ Y_{src} \\ Z_{src} \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} X_{alt} \\ Y_{alt} \\ Z_{alt} \end{bmatrix} = \begin{bmatrix} X_{w\_alt} & 0 & 0 \\ 0 & Y_{w\_alt} & 0 \\ 0 & 0 & Z_{w\_alt} \end{bmatrix} \begin{bmatrix} 1/X_{w\_src} & 0 & 0 \\ 0 & 1/Y_{w\_src} & 0 \\ 0 & 0 & 1/Z_{w\_src} \end{bmatrix} \begin{bmatrix} X_{src} \\ Y_{src} \\ Z_{src} \end{bmatrix} \tag{22}$$

$$\mathbf{c}_{alt} = diag(\mathbf{c}_{w\_alt})diag(\mathbf{c}_{w\_src})^{-1}\mathbf{c}_{src} \tag{23}$$

Similarly, substrate correction can also be used to derive reflectance factor aims for an alternate media (equation 24) in terms of reflectance measurement vectors.

$$\mathbf{r}_{alt} = diag(\mathbf{r}_{w\_alt})diag(\mathbf{r}_{w\_src})^{-1}\mathbf{r}_{src} \tag{24}$$

(Note: this also corresponds to spectral media relative operations in ICC.2 or iccMAX profiles).[4]

The last two terms of equation 24 represent a normalization of the reflectance to derive a normalized reflectance vector $\mathbf{n}_{src}$ that is independent of the media. This can be conceptualized physically as light being reflected by the medium and then through the colorant (ink) (see Figure 5) and is represented by equation 25. Rearranging the wavelength equation of the entries of the vector result in equation 26.

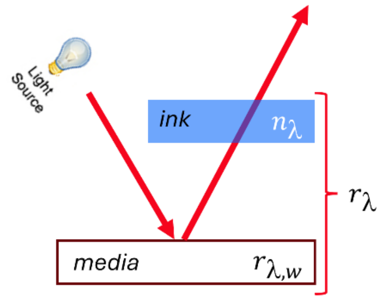

Figure 5 – Physical representation of equations 25 and 26

$$\mathbf{n}_{src} = \begin{bmatrix} {r_{1,src}}/{r_{1,w\_src}} \\ \vdots \\ {r_{m,src}}/{r_{m,w\_src}} \end{bmatrix} = diag\left(\mathbf{r}_{w_{src}}\right)^{-1}\mathbf{r}_{src} \tag{25}$$

where

$$n_\lambda = \frac{r_{\lambda,}}{r_{\lambda,w}}$$

$$r_\lambda = r_{\lambda,w} n_\lambda \tag{26}$$

Substituting Equation 25 into Equation 24 results in an equation that adjusts the normalized reflectance on a source media to estimate the reflectance on an alternate media.

$$\mathbf{r}_{\text{alt}} = diag\left(\mathbf{r}_{\text{w\_alt}}\right)\boldsymbol{n}_{src} \tag{27}$$

However, a better physical model would have the light first pass through the colorant and then reflected by the media back through the colorant as represented by Figure 6. Because the light passes through the colorant twice the media independent factor of the reflectance is represented as the square root of the normalized reflectance as shown in Equation 28.

$$s_\lambda = \sqrt{n_\lambda} = \sqrt{\frac{r_\lambda}{r_{\lambda,w}}} \tag{28}$$

This results in a re-formulation of the substrate corrected spectral estimation of Equation 27 in terms of the root normalization term (s_λ) from Equation 28 as a vector ($\boldsymbol{s}_{\text{src}}$) as follows:

$$\mathbf{r}_{\text{alt}} = diag(\mathbf{s}_{\text{src}})diag(\mathbf{r}_{\text{w\_alt}})\mathbf{s}_{\text{src}} \tag{29}$$

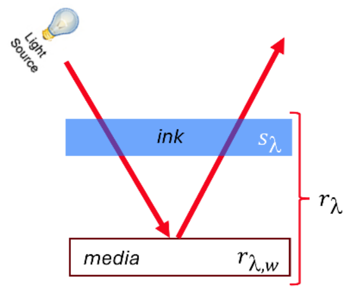

Figure 6 – Physical representation of equation 28

Fluorescence Substrate Correction (FSC) can be defined by restating Equation 29 in terms of matrices and replacing the diagonal of the reflectance of the alternate media with a fluorescence matrix for the alternate media ($\mathbf{F}_{\text{w\_alt}}$). This allows a fluorescence matrix ($\mathbf{F}_{\text{alt}}$) corresponding to a root normalized reflectance measurement from a non-fluorescing media to be defined relative to an alternate media with a fluorescence measurement matrix ($\mathbf{F}_{\text{w\_alt}}$) as shown in equation 30.

$$\mathbf{F}_{\text{alt}} = diag(\boldsymbol{s}_{\text{src,emitted}})\mathbf{F}_{\text{w\_alt}}diag(\boldsymbol{s}_{\text{src,excitation}}) \tag{30}$$

Note: With Equation 30 the root normalized vector ($\mathbf{s}_{\text{src}}$) may need to be expressed in different wavelength ranges to match the wavelengths of the excitation ($\boldsymbol{s}_{\text{src,excitation}}$) and emission ($\boldsymbol{s}_{\text{src,emitted}}$) wavelengths associated with the fluorescence matrix ($\mathbf{F}_{\text{w\_alt}}$) for the alternate media.

Applied example and discussion

Fluorescent Substrate Correction (FSC) as defined by Equation 30 was used with reflectance measurements of an IT8.7/4 chart and the fluorescence (Donaldson) matrix in Figure 4 to determine a set of spectral fluorescence matrices of an IT8.7/4 chart on a hypothetical media with an optical brightener with significant fluorescence. The spectral reflectances used for FSC were extended to cover the full excitation range of the media fluorescence matrix (see Figure 7).
Example fluorescent matrices for the IT8 primary and secondary colors are shown in Figure 8. The reflectance information is carried in the diagonal with off

diagonal values controlling fluorescence. The FSC maintains fluorescence from the media when the reflectance doesn't absorb all the near ultraviolet light.

A comparison was then made between the accuracy of using actual fluorescence measurements and using spectral reflectance measurements when fluorescence occurs.

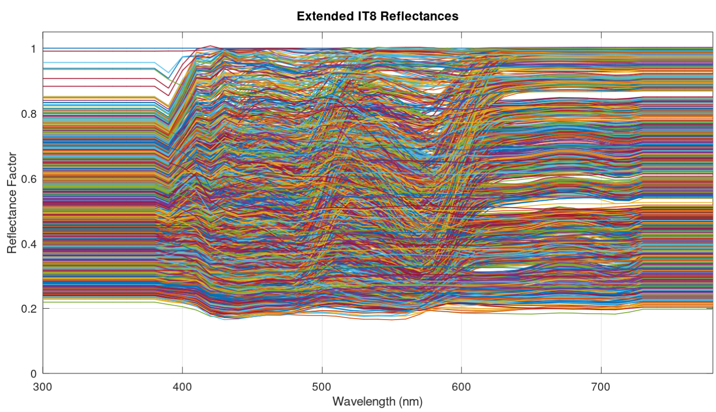

Figure 7 – Extended reflectance measurements of IT8 patches used for FSC

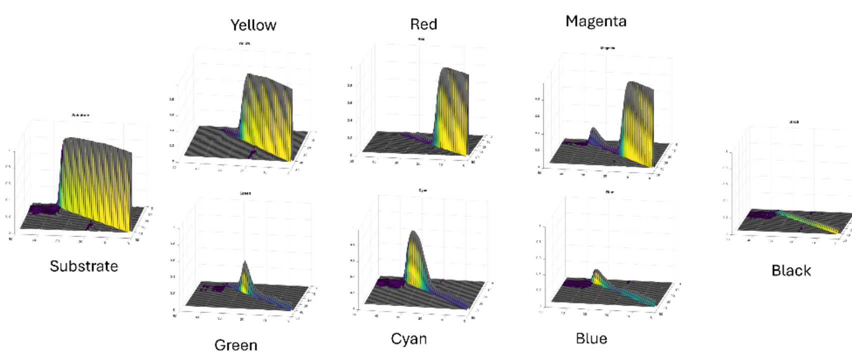

Figure 8 – Example estimated fluorescence matrices for IT8 primaries

The FSC fluorescence measurement matrices were applied with a virtual spectrophotometer model using Equation 31 based on Equation 17 with seven different measurement light sources ($\mathbf{w}_{\mathrm{meas}}$) to derive seven sets of spectral reflectance measurements of the IT8 patches.

$$\mathbf{r}_{\mathrm{i,meas}} = diag(\boldsymbol{w}_{\mathrm{meas}})^{-1}\mathbf{F}_{\mathrm{i}}\boldsymbol{w}_{\mathrm{meas}} \qquad (31)$$

The relative spectral power distributions of the seven measurement "light sources" can be found in Figure 9. It can be noted when comparing these power distributions that the "D50 no UV", C, and "White LED" SPDs have wavelength regions where there is zero relative power. This results in an undefined reflectance factor calculation for these wavelengths due to dividing by zero. In these cases, the measured reflectance factor was set to zero for wavelengths where the illuminant SPD was near zero.

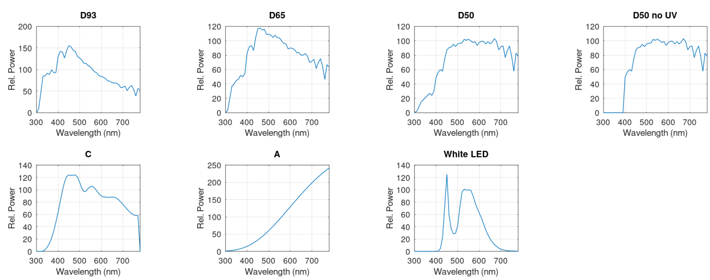

Figure 9 – Spectral Power Distributions (SPDs) of measurement light sources

The same set of seven light sources were then used as viewing light sources ($\mathbf{w}_{\text{view}}$) for determining colorimetry for each (i^{th}) patch of both the IT8 fluorescence matrices (Equation 32 based on Equation 11) and the seven sets of corresponding spectral reflectance measurements (Equation 33 based on Equation 12).

$$\mathbf{c}_{i,\text{view}} = \mathbf{CF}_i \mathbf{w}_{\text{view}} \tag{32}$$

$$\mathbf{c}_{i,\text{view,meas}} = \mathbf{C}diag(\mathbf{w}_{\text{view}})\mathbf{r}_{i,\text{meas}} \tag{33}$$

$$d_{i,\text{view,meas}} = \Delta E_{ab}\left(Lab\left(\mathbf{c}_{\text{i,view}}, view\right), Lab\left(\mathbf{c}_{\text{i,view,meas}}, view\right)\right) \tag{34}$$

Mean and maximum CIELAB color differences were determined by applying (Equation 34) between fluorescent measurements and reflectance measurements for each combination of measurement and viewing light sources.

Measurement Light Source	Viewing Light Source: D93	D65	D50	D50 no UV	C	A	White LED
D93	0.00	2.62	4.99	5.01	2.48	9.34	5.25
D65	2.64	0.00	2.38	2.40	0.77	6.83	4.61
D50	5.05	2.40	0.00	0.04	2.73	4.63	5.57
D50 no UV	5.23	2.49	0.04	0.00	2.82	4.70	5.51
C	2.55	0.74	2.78	2.80	0.00	7.15	4.14
A	9.92	7.17	4.79	4.77	7.34	0.00	9.25
White LED	3.43	3.47	5.16	5.16	3.12	9.35	0.05

Table 1 – Mean color differences between fluorescent measurements and reflectance measurements

Mean color difference results are shown in Table 1, and maximum color difference results are shown in Table 2.

Measurement Light Source	Viewing Light Source: D93	D65	D50	D50 no UV	C	A	White LED
D93	0.00	7.53	14.94	14.95	5.68	28.36	11.93
D65	7.33	0.00	7.19	7.20	3.33	20.29	14.83
D50	14.17	7.01	0.00	0.06	9.74	12.99	18.54
D50 no UV	13.67	6.77	0.06	0.00	9.39	12.56	18.55
C	5.70	3.21	9.54	9.55	0.00	21.48	14.59
A	25.76	19.03	12.56	12.54	21.03	0.00	29.41
White LED	10.19	13.76	18.00	17.99	13.48	28.81	0.08

Table 2 – Maximum color differences between fluorescent measurements and reflectance measurements

One might notice there are relatively small differences between the D50 and "D50 no UV" comparisons. The reason for these small differences is due to setting the reflectance factor to zero when the illumination had nearly zero relative power.

One conclusion from this virtual experiment is that the colorimetry for spectral reflectances (that involve fluorescence) under a desired light source/illuminant is only correct for the light source that was used in the spectrophotometer to obtain the measurement. Thus:

$$\mathbf{C}\mathbf{F}_i\mathbf{w}_{\text{view}} = \mathbf{C}\,diag(\mathbf{w}_{\text{view}})\mathbf{r}_{\text{F}_\text{i},\text{meas}} \tag{35}$$

only when the "view" and "meas" light sources are the same.

Thus, repurposing spectral reflectances to other light sources/illuminants results in errors in predicting the color appearance.

Correcting for Measurement Light Source

One question that resulted from this experiment was, "Can some form of correction be applied to spectral reflectance measurements to better predict the impact of fluorescence in the measured reflectance using a different light source in the measurement process?" Thus, can some matrix $\mathbf{M}_{\text{view,meas}}$ be found that predicts the measured reflectance with a change in the measurement light source as shown in Equation 36.

$$\mathbf{r}_{\text{F}_\text{i},\text{view}} = \mathbf{M}_{\text{view,meas}}\mathbf{r}_{\text{F}_\text{i},\text{meas}} \tag{36}$$

This can easily be accomplished using regression with a pseudo-inverse approach if sets of reflectance measurements using both light sources are available. This is demonstrated in the equations leading to Equation 37.

$$\boldsymbol{R}_{\text{view}} = [\boldsymbol{r}_{\text{F}_1,\text{view}} \quad \cdots \quad \boldsymbol{r}_{\text{F}_\text{n},\text{view}}], \boldsymbol{R}_{\text{meas}} = [\boldsymbol{r}_{\text{F}_1,\text{meas}} \quad \cdots \quad \boldsymbol{r}_{\text{F}_\text{n},\text{meas}}]$$

$$\mathbf{R}_{\text{view}} = \mathbf{M}_{\text{view,meas}}\mathbf{R}_{\text{meas}}$$

$$\mathbf{M}_{\text{view,meas}} = \mathbf{R}_{\text{view}} pinv(\mathbf{R}_{\text{meas}}) \quad (37)$$

Correction matrices were then determined between using Equation 37 with viewing and measurement light sources using the reflectance measurements determined in the previous section. An example of the correction matrix going from a measurement light source of illuminant A to a viewing light source of D93 is shown in Figure x.

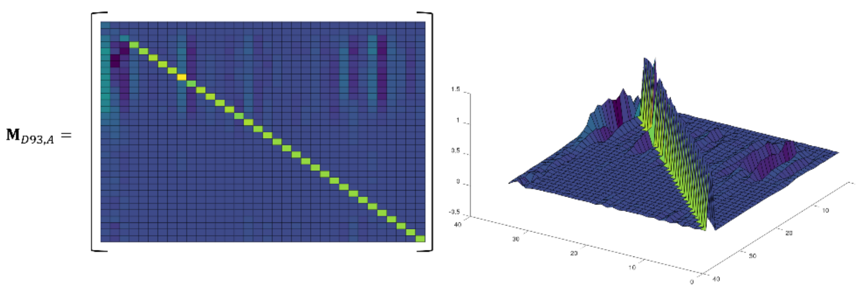

Figure 8 – Reflectance measurement correction matrix from Illuminant A to D93

Colorimetry for the corrected reflectances were then determined using Equation 37 and then compared to the fluorescent colorimetry from Equation 32 to determine the impact of using corrected reflectances.

$$\mathbf{c}_{i,\text{view,meas}} = \mathbf{C} diag(\mathbf{w}_{\text{view}})\mathbf{M}_{\text{view,meas}}\mathbf{r}_{i,\text{meas}} \quad (37)$$

Mean and maximum CIELAB color differences were determined by applying (Equation 34) between fluorescent measurements and reflectance measurements for each combination of measurement and viewing light sources.

		Viewing Light Source						
		D93	**D65**	**D50**	**D50 no UV**	**C**	**A**	**White LED**
Measurement Light Source	**D93**	0.00	0.02	0.04	0.07	0.05	0.06	0.08
	D65	0.03	0.00	0.02	0.05	0.03	0.04	0.07
	D50	0.05	0.02	0.00	0.03	0.02	0.02	0.06
	D50 no UV	0.08	0.05	0.03	0.00	0.02	0.02	0.02
	C	0.05	0.03	0.01	0.02	0.00	0.02	0.03
	A	0.07	0.04	0.02	0.02	0.01	0.00	0.06
	White LED	0.17	0.13	0.09	0.04	0.07	0.07	0.00

Table 3 – Mean color differences between fluorescent measurements and corrected reflectance measurements

Mean color difference results are shown in Table 3, and maximum color difference results are shown in Table 4.

Measurement Light Source	Viewing Light Source: D93	D65	D50	D50 no UV	C	A	White LED
D93	0.00	7.53	14.94	14.95	5.68	28.36	11.93
D65	7.33	0.00	7.19	7.20	3.33	20.29	14.83
D50	14.17	7.01	0.00	0.06	9.74	12.99	18.54
D50 no UV	13.67	6.77	0.06	0.00	9.39	12.56	18.55
C	5.70	3.21	9.54	9.55	0.00	21.48	14.59
A	25.76	19.03	12.56	12.54	21.03	0.00	29.41
White LED	10.19	13.76	18.00	17.99	13.48	28.81	0.08

Table 4 – Maximum color differences between fluorescent measurements and corrected reflectance measurements

This correction doesn't exactly predict the colorimetric results using spectral fluorescent measurements. However, it does provide improved prediction with dramatically less colorimetric error.

One significant challenge with this approach is that it requires having the reflectance measurements using the correct measurement light source in order to determine each correction matrix. In other words, the answers are needed before one can find a correction that approximates the answers, and this requires a correct understanding of the fluorescent behavior of what is being measured.

Conclusions

A discussion of the differences and challenges between measuring spectral reflectance and spectral fluorescence has been presented. A method of performing fluorescent substrate correction was presented that provides a means of understanding and quantifying the impacts of fluorescence in measuring spectral reflectance. It was shown that using measurements of spectral reflectance to determine colorimetry when fluorescence occurs as part of the measurement results in an incorrect determination of color appearance when the light source of the measurement is not the same as the light source that was used to obtain the measurement.

The extent of the error is directly related to the differences in fluorescence under the measurement and viewing light sources. The larger the difference of spectral power between measurement light sources in the fluorescent excitation wavelengths the larger the difference will be between calculated colorimetry and actual experienced colorimetry.

The impact of this should be considered when using spectral reflectance measurements for color management for viewing conditions that are significantly different than the lighting conditions used in the measurement process when there is fluorescence in the media or printing colorants. Repurposing spectral reflectances to other light sources/illuminants results in errors in predicting the color appearance.

It was shown that a correction of reflectances from one measurement light source to another is possible, but it requires knowing the reflectance measured under both light sources. Future research efforts could involve finding correlations with measurements to spectral fluorescent media modeling to allow for corrections to be applied to actual spectral reflectance measurements to get better prediction of color appearance with fluorescence under arbitrary light sources.

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